Magnetically-Tunable Microwave Filters Using Single-Crystal Yttrium-Iron-Garnet Resonators*

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Summary—**A new type of magnetically-tunable band-pass microwave filter that makes use of ferrimagnetic resonance in single-crystal yttrium iron garnet is presented. The 3-db bandwidth can be adjusted from about 6 Mc to 100 Mc at *X* band, and the center frequency can be tuned over a wide range of frequencies, by means of a varying dc field. A theoretical analysis of the operation and behavior of this type of filter is presented. Descriptions of single-resonator and two-resonator filters which can be tuned over the *X*-band frequency range are given and experimental data are presented showing their tuning range, insertion loss, and bandwidth.**

I. Introduction

With the increased use of continuously tunable microwave receivers, there has been a demand for a nonmechanically tuned, narrow-band filter which is tunable over a broad band of frequencies in the microwave frequency range. Previous attempts to solve this problem have included the use of cavities containtaining ferroelectrics,[1] "Varicaps" or voltage-tuned back-biased diodes,[2,3] and ferrites.[4–6] In general, tuning ranges of a few per cent are possible using these techniques while maintaining a high unloaded resonator $Q(Q_u)$.

The main difficulty with previous tuning techniques is high losses, which are greatly reduced by the use of the ferrimagnetic material, yttrium-iron-garnet (YIG). Single crystals of this material (now available commercially) show extremely low losses when used as the resonant elements in a band-pass (or band-reject) filter. The Q_u of this material considered as a resonator is typically between 2000 and 4000 at X-band frequencies and therefore compares favorably with transmission-line and hollow-cavity resonators.

This new approach to the problem of nonmechanical tuning makes use of the equivalence between a resonant circuit, such as an inductively coupled cavity or lumped-element series-resonant circuit, and an inductively coupled magnetic resonator biased with a dc magnetic field.

This equivalent circuit was first worked out analytically by Bloembergen and Pound[7] and was first applied by DeGrasse[8] to the design of an S-band limiter, using a single-crystal YIG resonator.

The tunability feature results from the fact that the resonant frequency is nearly a linear function of dc magnetic biasing field. This feature allows a very broad tuning range, limited mainly by the bandwidth of the microwave coupling structure employed.

The first part of this paper consists of a mathematical analysis of the equivalent circuit of a ferrimagnetic resonator coupled to external loads by means of loops, transmission lines, and waveguides. Analytical formulas for the external $Q(Q_e)$ of the ferrimagnetic resonator are given for loops, strip-transmission-lines and TE_{10} mode rectangular waveguide. A formula is given which relates the unloaded $Q(Q_u)$ of the ferrite resonator to the properties of the ferrite (damping time and gyromagnetic ratio).

The second part of this paper consists of descriptions of, and performance data taken on, two types of magnetically tunable filters operating in the X-band region and above. These are: 1) a single-resonator filter which has a tuning range from 7.0 kMc to 11.2 kMc, and 2) a two-resonator filter whose tuning range extends from 8.2 to 14.0 kMc.

II. Basic Principle of Operation

The basic idea of the magnetic resonance filter is illustrated in Fig. 1. Two coils have their axes at right angles to each other, and a small ferrite sample is placed at the intersection of the coil axes. When the sample is not magnetized, no power is transferred between the coils because the loop axes are perpendicular to each

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[1] W. J. Gemulla and R. D. Hall, "Ferroelectrics at microwave frequencies," *Microwave J.*, vol. 3, pp. 47–51; February, 1960.

[2] E. M. T. Jones, G. L. Matthaei, S. B. Cohn, and B. M. Schiffman, "Design Criteria for Microwave Filters and Coupling Structures," Stanford Research Inst., Menlo Park, Calif. Tech. Rept. 5, SRI Project 2326, Contract No. DA 36-039 SC-74862; March, 1959.

[3] A. Uhlir, Jr., "The potential of semiconductor diodes in high-frequency communications," Proc. IRE, vol. 46, pp. 1099–1115; June, 1958.

[4] G. R. Jones, J. C. Cacheris, and C. A. Morrison, "Magnetic tuning of resonant cavities and wideband frequency modulation of klystrons," Proc. IRE, vol. 44, pp. 1431–1438; October, 1956.

[5] C. E. Fay, "Ferrite-tuned resonant cavities," Proc. IRE, vol. 44, pp. 1446–1449; October, 1956.

[6] C. E. Nelson, "Ferrite-tunable microwave cavities and the introduction of a new reflectionless tunable microwave filter," Proc. IRE, vol. 44, pp. 1449–1455; October, 1956.

[7] N. Bloembergen and R. V. Pound, "Radiation damping in magnetic resonance experiments," *Phys. Rev.*, vol. 95, pp. 8–12; July 1, 1954.

[8] R. W. DeGrasse, "Low-loss gyromagnetic coupling through single crystal garnets," *J. Appl. Phys.*, Suppl. to vol. 30, pp. 155S–156S; April, 1959.

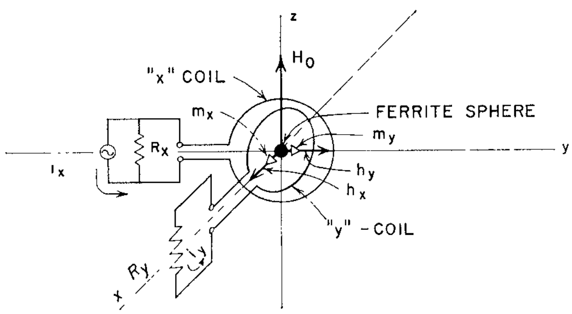

Fig. 1—Magnetic resonance filter.

other and there is no interaction with the ferrite. When a dc field H_0 is applied along the z axis, and an RF driving current $i_x e^{j\omega t}$ is applied to the terminals of the x coil, the magnetic moments of the electrons in the ferrite precess around the x axis, producing an RF magnetic moment along the y axis and inducing a voltage in the y circuit. The precession angle is largest and thus the induced voltage in the y-coil is maximum, at ferromagnetic resonance given by $\omega_0 = \mu_0 \gamma H_0$ for a spherical ferrite,[9] where γ is the gyromagnetic ratio and is very close to 1.759×10^{11} for electrons in most ferrites. The drop-off in response away from resonance is determined by the degree of coupling of the external loads R_x and R_y and the internal losses in the ferrite. It is shown in Section III that the equivalent circuit of this device is a resonant circuit, inductively coupled to the output lines or loads. This circuit is also a gyrator; the phase shift in one direction through the circuit differs from the phase shift in the other direction by 180°.

One requirement on the resonator in a low-insertion-loss, narrow-band-pass filter is that Q_u be high. Although it is necessary to specify the application in more detail in order to specify what exactly is meant by "high-Q_u," it may be stated in general that Q_u's of several hundred or higher are useful for filter design. Q_u's of this order are obtained at microwave frequencies with single-crystal-yttrium-iron-garnet.

The second requirement to obtain low insertion loss in a narrow-band, band-pass filter is that the external Q, Q_e of the resonators be considerably lower than Q_u. It was not apparent from previous work by DeGrasse[8] that sufficiently low Q_e could be obtained over a wide tuning range. It is shown below that sufficient coupling can actually be obtained without the use of the narrow-band impedance matching employed by DeGrasse.[8]

Two other characteristics of single crystal ferrimagnetic resonators must be considered in designing a magnetically tunable filter: 1) magnetocrystalline anisotropy, and 2) the higher-order magnetostatic modes. Their effects on filter characteristics and design are discussed below.

[9] These quantities are defined in Section III.

III. Analysis and Measurement of Coupling Characteristics and Equivalent Circuits of Ferrite Resonators in Lumped-Element Circuits, Transmission Lines, and Waveguides

In the following analysis, the ferrite resonator is assumed to be small enough that the RF field is substantially uniform throughout the volume of the sample and therefore that higher-order modes are not excited. Also magnetocrystalline anisotropy is neglected. In this case, with the dc magnetic field H_0 applied along the z-axis, the RF magnetizations in the x and y directions are given by the following relations:[10]

$$m_x = \chi_{xx}^e h_x + \chi_{xy}^e h_y, \quad (1a)$$

$$m_y = \chi_{yx}^e h_x + \chi_{yy}^e h_y, \quad (1b)$$

where

$m_x = x$ component of RF magnetization within ferrite,

$m_y = y$ component of RF magnetization within ferrite,

$h_x = x$ component of applied RF field,

$h_y = y$ component of applied RF field,

$\chi_{xx}^e = \chi_{yy}^e = x-x(y-y)$ component of effective RF tensor, susceptibility,[11]

$\chi_{xy}^e = -\chi_{yx}^e = x-y(y-x)$ component of effective RF tensor susceptibility.

Flammer[12] has developed analytical formulas for the effective susceptibility that include the loss in the ferrite. These expressions were derived using the Bloch-Bloembergen[13] formulation of the equations of motion of magnetization, including the effect of the shape-dependent demagnetizing factors N_x, N_y, and N_z for a

[10] MKS units are used here and throughout this paper.

[11] The effective or external susceptibility is the ratio of the RF magnetic moment to the *applied* RF magnetic field (not the RF magnetic field inside the ferrite).

[12] C. Flammer, "Resonance Phenomena in Ferrites," Stanford Research Inst., Menlo Park, Calif. unpublished memorandum; 1956. Stanford plans to publish this memorandum in the near future.

[13] N. Bloembergen, "Magnetic resonance in ferrites," Proc. IRE, vol. 44, pp. 1259–1269; October, 1956. It has been pointed out by Flammer that (5) in this reference contains an error. The term $-M_0/\gamma$ should be deleted.

general ellipsoidal sample.[14] These formulas for the effective tensor susceptibility are

$$\chi_{xx}^e = \frac{\omega_m\left[\omega_0 + (N_y - N_z)\omega_m + \frac{j\omega/\tau}{\omega_0 - N_z\omega_m}\right] + \frac{\omega_m}{(\omega_0 - N_z\omega_m)^2}[\omega_0 + (N_y - N_z)\omega_m]\cdot\frac{1}{\tau^2}}{\left\{\begin{array}{l}\left[\omega_0 + (N_y - N_z)\omega_m + \frac{j\omega/\tau}{\omega_0 - N_z\omega_m}\right]\left[\omega_0 + (N_x - N_z)\omega_m + \frac{j\omega/\tau}{\omega_0 - N_z\omega_m}\right] - \omega^2 \\ + \frac{1}{(\omega_0 - N_z\omega_m)^2\tau^2}\cdot[\omega_0 + (N_x - N_z)\omega_m][\omega_0 + (N_y - N_z)\omega_m]\end{array}\right\}}, \tag{2a}$$

$$\chi_{yy}^e = \frac{\omega_m\omega_0 + (N_x - N_z)\omega_m + \frac{j\omega/\tau}{(\omega_0 - N_z\omega_m)^2} + \frac{\omega_m}{(\omega_0 - N_z\omega_m)^2}[\omega_0 + (N_x - N_z)\omega_m]\frac{1}{\tau^2}}{\left\{\begin{array}{l}\left[\omega_0 + (N_x - N_z)\omega_m + \frac{j\omega/\tau}{\omega_0 - N_z\omega_m}\right]\left[\omega_0 + (N_y - N_z)\omega_m + j\frac{\omega/\tau}{\omega_0 - N_z\omega_m}\right] - \omega^2 \\ + \frac{1}{(\omega_0 - N_z\omega_m)^2\tau^2}[\omega_0 + (N_x - N_z)\omega_m][\omega_0 + (N_y - N_z)\omega_m]\end{array}\right\}}, \tag{2b}$$

$$\chi_{xy}^e = -\chi_{yx}^e = \frac{-j\omega_m\omega}{\left\{\begin{array}{l}\left[\omega_0 + (N_x - N_z)\omega_m + \frac{j\omega/\tau}{\omega_0 - N_z\omega_m}\right]\left[\omega_0 + (N_y - N_z)\omega_m + \frac{j\omega/\tau}{\omega_0 - N_z\omega_m}\right] - \omega^2 \\ + \frac{1}{(\omega_0 - N_z\omega_m)^2\tau^2}[\omega_0 + (N_x - N_z)\omega_m][\omega_0 + (N_y - N_z)\omega_m]\end{array}\right\}}, \tag{2c}$$

in which

$\omega_m = \mu_0\gamma M_0 = \gamma_0 M_0$,
$\omega_0 = \mu_0\gamma H_0$,
γ = gyromagnetic ratio $= g(e/2m)$,
N_x, N_y, N_z = effective demagnetizing factors in x, y, and z directions,
τ = Bloch-Bloembergen phenomenological relaxation time,
g = Landé "g" factor $\cong 2.00$ for electrons in most ferrites,
e/m = ratio of charge e to mass m of electron, $= 1.759 \times 10^{+11}$ coulombs/kg,
M_0 = saturation magnetization in amperes per meter,
μ_0 = intrinsic permeability of free space $= 1.256 \times 10^{-6}$ henries per meter,
H_0 = applied dc field in amperes per meter.

For a spherical geometry, $N_x = N_y = N_z = \frac{1}{3}$; and (2) becomes after some algebraic reduction and elimination of terms of order $1/\tau^2$ or smaller,

$$\chi_{xx}^e = \chi_{yy}^e = \frac{\omega_0\omega_m}{\omega_0^2 - \omega^2 + \frac{2j\omega}{\tau}\left(1 + \frac{\omega_m/3}{\omega_0 - \omega_m/3}\right)} \tag{3a}$$

$$\chi_{xy}^e = -\chi_{yx}^e = \frac{-j\omega_m\omega}{\omega_0^2 - \omega^2 + \frac{2j\omega}{\tau}\left(1 + \frac{\omega_m/3}{\omega_0 - \omega_m/3}\right)}. \tag{3b}$$

[14] C. Kittel, "On the theory of ferromagnetic resonance absorption," *Phys. Rev.*, vol. 73, pp. 155–161; January 15, 1948.

The complete detailed equivalent circuit[15] of the ferrimagnetic resonator located at the intersection of the two loops is shown in Fig. 2. It consists of a resonant circuit coupled inductively to the signal source i_x, to the source impedance R_x, and to the load impedance R_y. In addition to the series-resonant circuit, there is a nonreciprocal phase shifter which produces, with the coil arrangement shown in Fig. 1, an additional *negative* 90° phase shift in the output voltage across R_y. With the signal generator connected to the y terminals an additional *positive* 90° phase shift is produced across R_x.

The complete analytical formulation of the circuit representation of the ferrimagnetic resonator has been carried out for the following cases:

1) Ferrimagnetic resonator located at the intersection of the axes of the two orthogonal loops shown in Fig. 1.
2) Resonator located at a position of an equivalent short circuit along a short-circuited two-wire TEM-mode transmission line, shown in Fig. 3(b).
3) Same as case 2 in a strip-center-conductor line.
4) Resonator located at a position of an equivalent short circuit at the center of a short-circuited TE_{10}-mode waveguide.

[15] P. S. Carter, Jr. and G. L. Matthaei, "Design Criteria for Microwave Filters and Coupling Structures," Stanford Research Inst., Menlo Park, Calif. Tech. Rept. 8, SRI Project 2326, Contract No. DA 36-039 SC-74862; September, 1959.

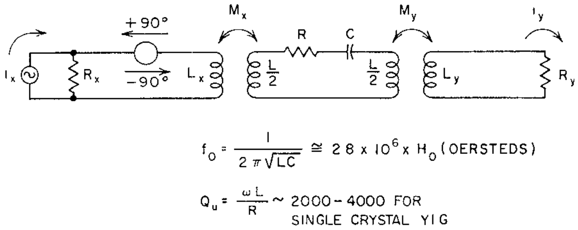

Fig 2—Equivalent circuit of magnetic resonance filter.

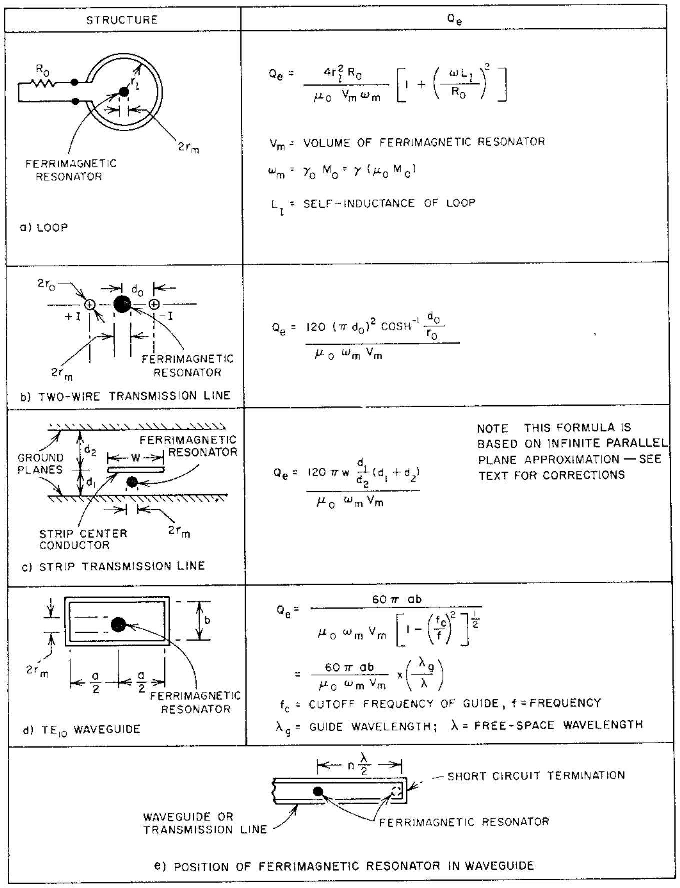

Fig. 3—Q_e of ferrimagnetic resonator in several waveguide structures.

In each case the complete equivalent circuit of the ferrimagnetic resonator coupled to the external circuit is expressed in terms of the unloaded Q_u of the resonator and the coupled or external Q_e.[16]

The unloaded Q_u of the ferrimagnetic resonator is a property of the ferrimagnetic material and is independent of the external circuit.[17] An analytical formula for Q_u derived from the equivalent circuit is given by[15]

$$Q_u = (\omega_0 - N_z\omega_m)\tau/2 = \frac{H_0 - N_z M_0}{\Delta H^i}, \tag{4}$$

[16] C. G. Montgomery, R. H. Dicke, and E. M. Purcell, "Principles of Microwave Circuits," M.I.T., Rad. Lab. Ser. No. 8, McGraw-Hill Book Co., Inc., New York, N. Y., pp. 228–239; 1948.

[17] This is true for the essentially unperturbed resonator located a sufficient distance from the boundaries of the waveguide. For close wall-to-resonator spacings, resistive losses due to image currents in the waveguide walls and the detuning effects of these images are significant to the resonance characteristics.

where ΔH^i is the line width given by

$$\Delta H^i = \frac{2}{\gamma_0 \tau}.$$

Eq. (4) predicts low values of unloaded Q_u for low resonant frequencies, and also predicts, in particular, that Q_u will reach zero at value of dc field and RF frequency given by $\omega_0 = N_z\omega_m$. For a spherical YIG resonator, this low cutoff frequency occurs at $f_0 = \omega_m/3\times 2\pi = 1670$ Mc. Measurements made of Q_u in the frequency range of this cutoff support this theory.[18]

Eq. (4) predicts that Q_u depends on $N_z\omega_m$. In particular, a long-rod geometry with the dc magnetic field along the axis of the rod (so that $N_z \approx 0$) should yield a maximum value of Q_u. This effect of the shape on Q_u has not yet been measured. Finally, (4) shows that, for a constant relaxation time τ, Q_u is higher in the case of a lower saturation magnetization material. For frequencies below around 2000 Mc, a low-M_0, narrow-linewidth material is desirable. This requirement is discussed in Section VII.

The external or coupled Q_e depends upon the volume of the ferrimagnetic resonator V_m, its saturation magnetization M_0, and the type and geometry of the coupling structure. Formulas for Q_e have been derived for the four cases listed. The resulting formulas, tabulated in Fig. 3, may be applied to the calculation of the Q_e of the uniform precessional mode of any ellipsoidally shaped magnetic resonator (rods, disks, etc).

The formulas for Q_e given in Fig. 3 were derived using two different methods: The first, which employs the reciprocity theorem,[15] is valid for lumped-parameter coupling devices such as loops, and for TEM-mode transmission lines. The equation for Q_e of the magnetic resonator in the TE_{10}-mode rectangular waveguide [Fig. 3(d)] was derived using a more general coupling formula discussed by Slater.[19] For the waveguide case, a *linearly-polarized* magnetic moment was assumed. The RF magnetic moment of a magnetic resonator is in general *elliptically* polarized, therefore, both perpendicular components of RF magnetic moment can interact with an elliptically polarized waveguide field. The formula for Q_e given above is thus valid only when the magnetic sample is located in the linearly-polarized field at the center of the guide.[20]

[18] P. S. Carter, Jr. and C. Flammer, "Unloaded Q of single crystal garnet resonator as a function of frequency," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-8 (Correspondence), pp. 570–571; September, 1960.

[19] J. C. Slater, "Microwave Electronics," D. Van Nostrand Co., Inc., New York, N. Y., ch. 7, p. 150; 1950.

[20] Recently while this paper was being revised, the author was made aware of a more general scattering formula which is applicable in the case where the ellipsoidal sample is at an arbitrary position in the guide. See H. J. Shaw and L. K. Anderson, "Ferrimagnetic resonance in ferrites," in "Interaction of Microwaves with Matter," Microwave Laboratory, Stanford University, Stanford, Calif., Tech. Status Rept. No. 9, Section 1, Contract AF 49(638)-415; May 1–July 31, 1960.

Measurements were made of the Q_e of single-crystal YIG resonators in strip transmission line and TE_{10}-mode rectangular waveguide, and the results of these measurements are shown in Figs. 4 and 5 Fig. 4 shows measurements made on spherical single-crystal YIG resonators of various diameters mounted on a wafer of polyfoam in a section of one-quarter height, standard width, X-band, rectangular waveguide. The waveguide was short-circuited and the YIG sphere was mounted one-half wavelength from the short circuit. The measurements were made in this configuration at 10.0 kMc, using the impedance method described by Ginzton.[21] The solid curve represents the theoretical formula for Q_e. Close agreement between the experimental and theoretical values is evident. When resonator diameters greater than about 0.070 inch were used, measurement accuracy was diminished by the high degree of coupling between the external circuit and the higher-order magnetostatic modes. The effect of the higher-order modes on filter performance is discussed in Section IV.

Fig. 5 shows measurements of Q_e of a strip-transmission-line coupling circuit. The strip-transmission-line configuration (Fig. 5) has a 0.500-inch-wide 0.020-inch-thick center conductor spaced 0.135 inch from the outer conductor on both sides of the center conductor. The inside width of the outer conductor is 1.00 inch. Q_e was measured at a resonant frequency, $f_0=3000$ Mc. In these measurements the YIG resonator was mounted close to the short-circuited end of the strip transmission line—the position occupied by the resonator in a magnetically tunable filter where a broad tuning range is desired—since this remains a position of nearly maximum magnetic field as the frequency is varied. The minimum distance between the sphere and the short-circuited end is prescribed by the maximum allowable coupling to higher-order modes. With the resonator touching the short-circuit, appreciable coupling to higher-order modes occurs. In the case shown in Fig. 5 with 0.125-inch spacing between the resonator and the end wall, experiment showed that the coupling to higher-order modes was very loose, *i.e.*, $Q_e > 15{,}000$, for the most strongly coupled higher-order mode. The coupling to the uniform precession resonance at 3000 Mc was not measurably decreased by this 0.125-inch spacing.

The theoretical values for Q_e of the strip transmission line are shown in Fig. 5 as the solid-line curve. This curve was calculated using the formula given in Fig. 3(c), which is very accurate where $w \gg d_1$ and d_2, but not quite so accurate where $w=0.500$ inch and $d_1=d_2=0.135$ inch (as in Fig. 5). An empirical correction was applied to improve the accuracy of this formula by replacing w in Fig. 3(c) by a slightly larger value, w', which takes into account the fringing of the field at the edge of the strip center conductor. This corrected width w' was obtained as follows: the characteristic impedance Z_0' of the actual configuration shown in Fig. 5 was obtained from an accurate formula[22] which includes the effect of the finite thickness of the center conductor and the fringing of the field at the two edges. This value of characteristic impedance Z_0' was then used to compute an effective strip width w', based on the parallel-plane formula, *i.e.*,

$$w' = \frac{\eta_0}{Z_0'} \frac{d_1 d_2}{d_1 + d_2},$$

again using the actual conductor spacings d_1 and d_2, where $\eta_0 = 377$ ohms.

The agreement between the measured and theoretical values of Q_e was quite good (Fig. 5). The measured values of Q_e differ appreciably from the calculated values only for the largest values of YIG sphere diameter

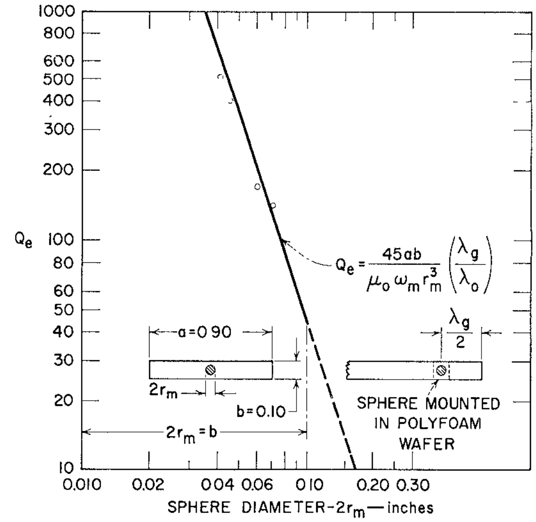

Fig. 4—Theoretical and experimental Q_e of YIG resonators in reduced-height (one-quarter), standard-width X-band waveguide at 10,000 Mc.

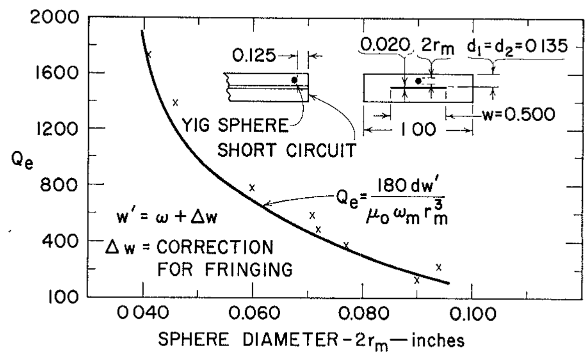

Fig. 5—Theoretical and experimental values of Q_e of YIG resonator using stripline coupling at 3000 Mc.

[21] E. L. Ginzton, "Microwave Measurements," McGraw-Hill Book Co., Inc., New York, N. Y., ch. 9, pp. 405–417; 1957.

[22] S. B. Cohn, "Problems in strip transmission lines," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-3, pp. 119–126; March, 1955.

(around 0.090 inch). For these largest-sphere diameters the measured value of Q_e is also sensitive to the exact location of the sphere with respect to the conducting walls. This shows that the boundary conditions imposed on the motion of the magnetization by the conducting surfaces of the strip transmission line affect the coupling appreciably.

The strip transmission line offers more design flexibility than the waveguide. Examination of the formula in Fig. 3(c) for the Q_e of a strip transmission line shows that it is possible to adjust both the characteristic impedance of the line and Q_e independently, by adjusting the strip outer conductor width w and the ground-plane spacings d_1 and d_2.

IV. Development of a Single-Resonator Tunable Filter

A single-resonator tunable filter using waveguide coupling was constructed which is tunable over the X-band frequency range from 7.00 kMc to 11.00 kMc. The coupling arrangement (Fig. 6) is analogous to the crossed loops in Fig. 1. The single-crystal YIG sphere is mounted on a styrofoam support in the open iris region common to both the input and output waveguides. The depth of this region of interpenetration, as well as the guide height and resonator size, can be varied to control the coupling between the guides and the resonator. The bias field is applied along the common axis of the guides.

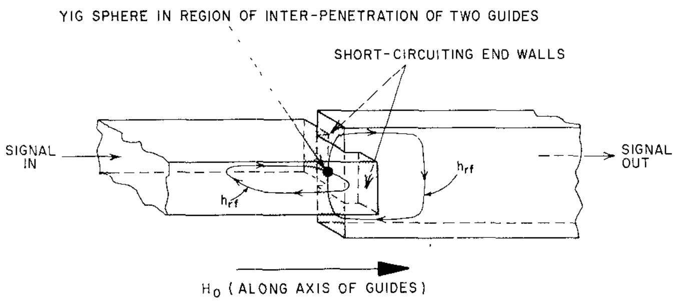

Fig. 6—Coupling principle of single-resonator waveguide filter.

Figs. 7 and 8 show the construction of the experimental single-resonator filter employing a standard-width X-band waveguide. The half-height ($b=0.200$ inch) coupling sections used in this experimental filter are followed by E-plane circular bends (mean radius $=0.300$ inch). These E-plane bends permit the use of a "c" type electromagnet for biasing along the axis of transmission of the guide. Immediately following the E-plane bends are 2.20-inch-long straight tapers joining the half-height sections to the full-height X-band input and output guides. The interpenetration of the two guides was accomplished by milling out 0.035-inch-deep slots in the face of the opposite section, as shown in Figs. 7 and 8.

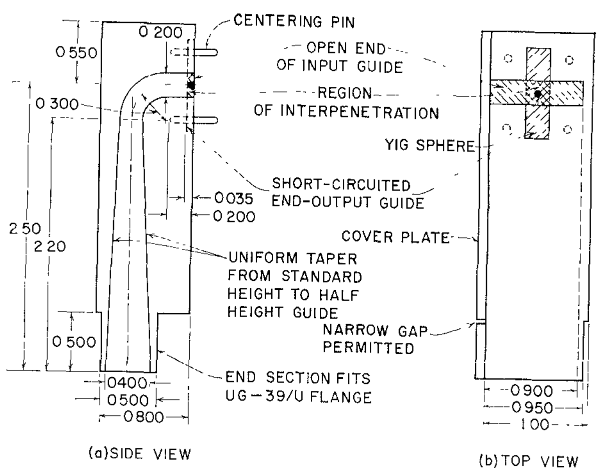

Fig. 7—Construction of half-section of single-resonator waveguide filter.

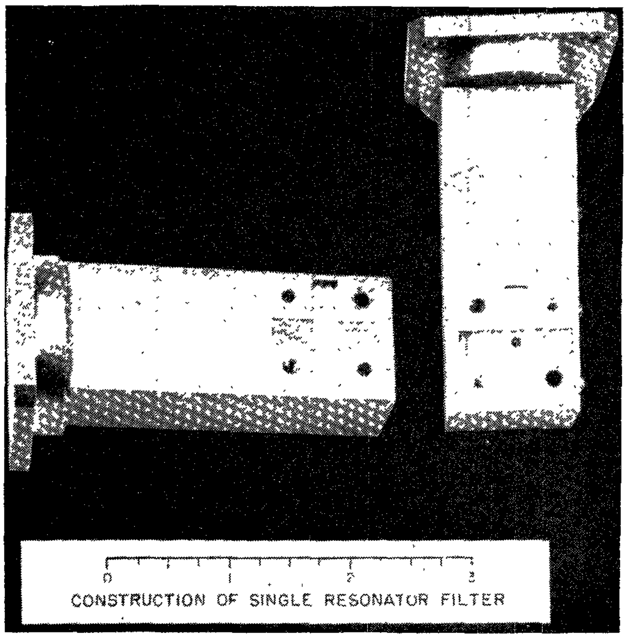

Fig. 8—Experimental single-resonator filter.

The measured responses when this single-resonator filter was tuned to resonate successively at 8.20, 9.00, 10.00, and 11.00 kMc are shown in Fig. 9. Table I shows the measured insertion losses at these frequencies and also at 7.00 kMc, 3-db bandwidths, and the peak response of the strongest subsidiary mode, and the values of Q_u calculated from the insertion loss and bandwidth data (using equations given by Ginzton[23]).

A prominent feature of this single-resonator filter response is the presence of many subsidiary responses which are due to higher-order magnetostatic resonances within the ferrimagnetic sample. These resonances, which were first analyzed by Walker,[24] result from the

[23] Ginzton, *op. cit.*, pp. 403–405.

[24] L. R. Walker, "Magnetostatic modes in ferromagnetic resonance," *Phys. Rev.*, vol. 105; January 15, 1957.

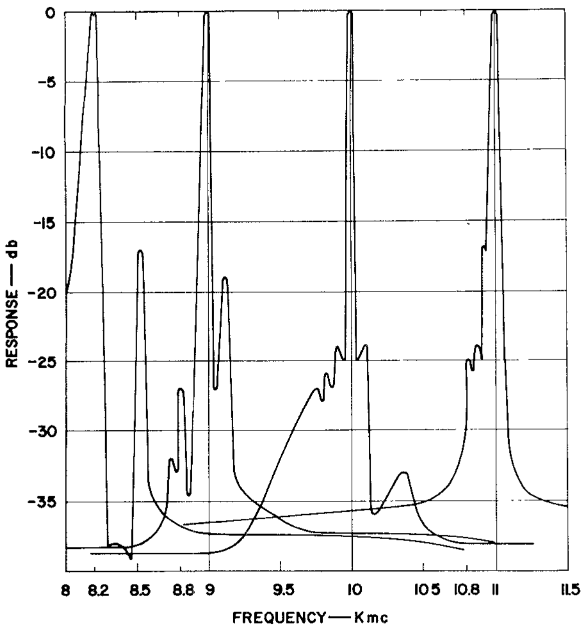

Fig. 9—Single-resonator filter response curves.

TABLE I
PERFORMANCE OF SINGLE-RESONATOR FILTER

f(kMc)	Insertion Loss at Resonance (db)	3-db Bandwidth (Mc)	Q_u	Strongest Magnetostatic Mode Response (db Below Transmission at Main Resonance)
7.00	2.0	not measured	—	—
8.20	1.5	14	3600	−17
9.00	1.2	20	3400	−18
10.00	1.3	25	3200	−23
11.00	1.1	28	3200	−17

nonuniform motion of the magnetization within the ferrimagnetic sample and the resulting dipolar interactions between the magnetic moments. Walker showed that the resonances are characterized by: 1) independence of the size of the sample, *i.e.*, they depend on the shape only, and 2) occurrence only at frequencies given by

$$\omega_0 + \gamma_0 \frac{M_0}{2} \geq \omega \geq \omega_0,$$

where

$$\omega_0 = \gamma_0 H_0.$$

Thus the resonant frequencies of the nonuniform resonances lie somewhere within a spectrum which is $\gamma_0 M_0/2$ cycles wide. For yttrium-iron-garnet in which the saturation magnetization M_0 is approximately 1395×10^2 amperes per meter, the width of this frequency band is 2450 Mc. Coupling to these nonuniform modes occurs when the RF coupling field is nonuniform over the dimensions of the sample. The maximum allowable response to higher-order modes thus dictates the largest-ferrimagnetic resonator that can be used in a specific coupling circuit. In practice, this restriction limits the diameter of the spherical resonator used in the present X-band single-resonator filter to less than 0.100 inch.

The spurious responses due to the higher-order magnetostatic modes are greatly reduced by the use of two or more ferrimagnetic resonators coupled directly together by means of their external RF fields. This is described in detail in Section VI.

A characteristic of this single-resonator filter response using TE_{10}-mode waveguide coupling is the increase of the bandwidth and the decrease of the insertion loss as the center frequency is raised. This increase in bandwidth, and decrease in insertion loss, is due to the decrease of the external Q_e of the waveguide coupling circuit as the frequency increases. Variation of the bandwidth over the tuning range is minimized by the use of a constant characteristic impedance coupling circuit, such as the TEM-mode strip transmission line shown in Fig. 3(c).

Another important aspect of the behavior of a YIG resonator is the saturation occurring at high power levels. This effect is explained in terms of the nonlinear coupling between the uniform mode and the short-wavelength "spin waves," and has been investigated and explained by Suhl.[25,26] This saturation effect was found by experiment to limit the peak power of the X-band single-resonator filter to less than about 15 watts. This effect has been discussed further and experimental results have been presented by Carter and Matthaei.[27] Considerably lower saturation thresholds occur at lower frequencies in the S-band range, both in theory[26] and in experiment.[8]

It has been shown[28] that Q_u is critically dependent on the achievement of a high surface polish on a yttrium-iron-garnet sphere. The spheres used here were ground and polished by a simple tumbling procedure which has been described by Bond.[29] The spheres were ground to shape from the raw crystals by tumbling, using No. 120 grade silicon carbide abrasive paper. They were then ground and polished using this same tumbling technique with successively finer grades of abrasive grinding paper and powder, as follows:

silicon carbide, No. 360, No. 600, No. 900,
emery polishing paper, No. 2/0 and No. 4/0,
Linde fine abrasive, No. B-5125.

[25] H. Suhl, "The non-linear behavior of ferrites at high signal levels," PROC. IRE, vol. 44, pp. 1270–1284; October, 1956.

[26] H. Suhl, "The theory of ferromagnetic resonance at high signal powers," *J. Phys. Chem. Solids*, vol. 1, pp. 209–217; January, 1957.

[27] P. S. Carter, G. L. Matthaei, "Design Criteria for Microwave Filters and Coupling Structures," Stanford Research Inst., Menlo Park, Calif., Final Rept. SRI Project 2326, Contract No. DA 36-039 SC-74862; January, 1961.

[28] R. C. LeCraw, E. G. Spencer, and C. S. Porter, "Ferromagnetic resonance line in yttrium-iron-garnet single crystals," *Phys. Rev.*, vol. 110, pp. 1311–1313; June 15, 1958.

[29] W. L. Bond, "Making small spheres," *Rev. Sci. Instr.*, vol. 22, pp. 344–345; May, 1951.

V. Development of a Two-Resonator Filter

Fig. 10 shows the application of two garnet spheres to form a two-resonator filter. Coupling between the resonators is provided by the long-slot iris in a 0.010-inch-thick conducting wall separating the input and output waveguides.[30] The long vertical coupling slot permits the vertical components of RF magnetic moment to be coupled together through their external RF fields, and prevents the horizontal magnetic fields of the waveguides and horizontal RF magnetic moments from being coupled together appreciably.

The two spheres are synchronously tuned by mounting them in dielectric capsules so that they can turn in any direction. Application of a dc biasing magnetic field results in a torque that causes one of the "easy axes" of magnetization to align itself along the dc magnetic field. Thus, detuning that would be caused by magnetocrystalline anisotropy[31] is easily eliminated.

Fig. 11 shows the disassembled two-resonator filter with the coupling iris, and the garnet spheres in 0.080-inch-diameter dielectric capsules. The capsules mounted on polystyrene rods are adjusted to vary the spacing of the spheres. Two 90° *E*-plane bends make it possible to use a "*c*"-type electromagnet.

In this experimental model, the input and output guides are standard-height *X*-band guides. Here, as for the single-resonator filter, the guide height or the sphere diameter can be varied to obtain the desired filter response. The two YIG spheres used in this model were 0.064 inch and 0.060 inch in diameter.

Fig. 12 shows the measured responses of the two-resonator filter tuned to 8.23 kMc, 9.49 kMc, 10.99 kMc, 12.00 kMc, 13.00 kMc, and 14.25 kMc. The initial adjustment of the filter was made at 9.49 kMc by varying the coupling slot dimensions and the spacing of the resonators to give the slightly overcoupled response with greater than 35-db insertion loss at frequencies far removed from the 9.49-kMc center frequency. This last requirement, *i.e.*, on rejection loss at frequencies far removed from resonance, limits the maximum width of the coupling slot, since the coupling between the guides in the absence of the garnet resonators is proportional to the width of the slot. The isolation obtained in this way can be calculated from the slot dimensions.[32,33] The slot dimensions used in this filter were 0.070 inch by 0.300 inch and were determined experimentally to give greater than 35-db off-channel rejection. The spacing between the centers of the YIG spheres was 0.164 inch. It was found that almost any desired degree of overcoupling could be obtained by closer spacing of the gar-

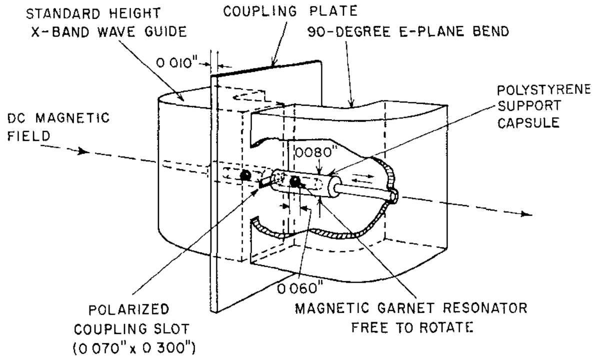

Fig. 10—Internal construction of two-resonator tunable filter.

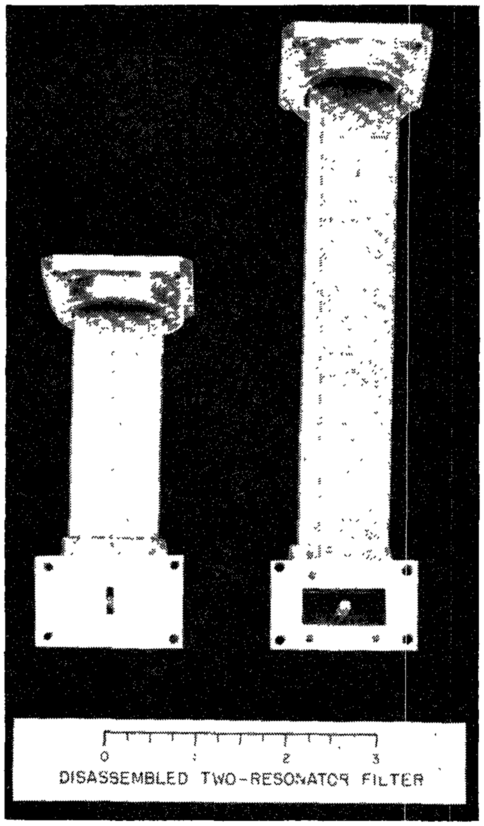

Fig. 11—Disassembled two-resonator filter.

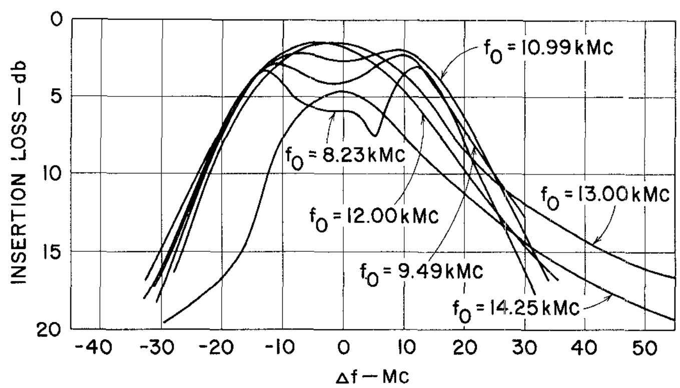

Fig. 12—Measured frequency responses of two-resonator filter.

[30] The use of polarized slot coupling was suggested by E. M. T. Jones of Stanford Research Inst., Menlo Park, Calif.

[31] C. Kittel, "On the theory of ferromagnetic resonance absorption," *Phys. Rev.*, vol. 73, pp. 155–161; January 15, 1948.

[32] H. A. Bethe, "Lumped Constants for Small Irises," M.I.T. Rad. Lab., Cambridge, Mass., Rept. No. 43-22; March 24, 1943.

[33] S. B. Cohn, "Microwave coupling by large apertures," Proc. IRE, vol. 40, pp. 696–699; June, 1952.

net spheres. The 3-db and 10-db bandwidths are nearly constant at about 32 Mc and 56 Mc, respectively, as the center frequency is changed with varying frequency.

The trends observed in Fig. 12 are: 1) a decrease of the degree of over-coupling with increasing frequency, 2) an approximately constant bandwidth throughout the tuning range, and 3) a decrease of insertion loss with increasing frequency. These trends are attributed to the same effect which caused the increase in bandwidth of the single-resonator filter, *i.e.*, the increase, with increasing frequency, of the coupling between the resonators and the waveguides.

The maximum tuning range of this experimental two-resonator filter appears to be limited by the leakage at 15.5 kMc through the coupling slot. This is shown in Fig. 13, which is a plot over a broadband of the response of the filter tuned to 12 kMc. This leakage is due to resonance of the slot at 15.5 kMc; it is then one-half wavelength long, and it couples energy between the two guides since it is not exactly parallel to the electric field of the waveguide. Evidently this slot response might be reduced by more accurate construction—this slot was cut by hand. Also, a shorter, wider slot might give nearly the same response and insertion loss, while moving the half-wave slot resonance to a higher frequency.

Fig. 14 shows the response of the two-resonator filter down to −30 db below the peak response, with the center frequency at 11.0 kMc. No spurious responses are evident here, as they were with the single resonator filter. This greater rejection of the higher-order modes is explained by the fact that the RF fields of the higher-order modes decrease very rapidly with distance away from the spheres, more rapidly than do the RF fields of the uniform mode. The higher-order modes of the two resonators are therefore coupled together very weakly through their external RF fields, making the insertion loss of these modes very high compared to that of the uniform mode. The absence of spurious responses in the two-resonator filter is an important advantage of this type of filter over the single-resonator type.

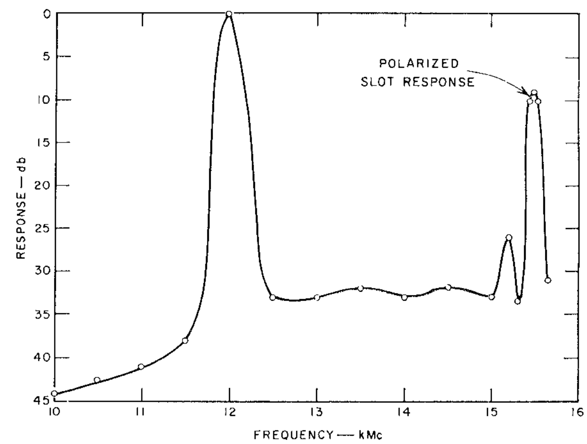

Fig. 13—Response of two-resonator filter tuned to 12 kMc showing polarized slot leakage at 15 kMc.

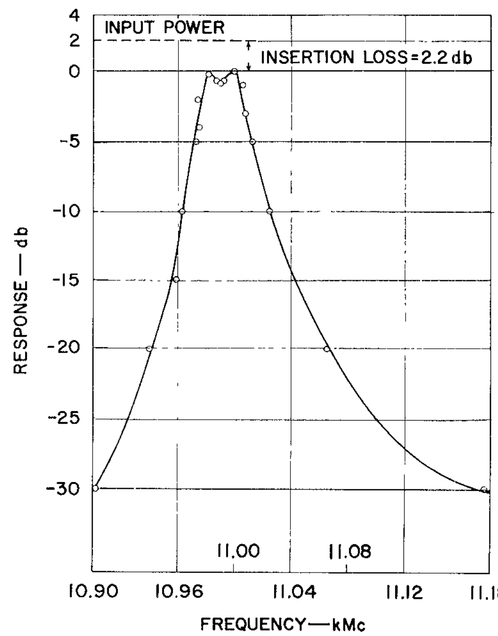

Fig. 14—Response of two-resonator filter tuned to 11 kMc.

VI. Future Developments

A number of useful extensions can be made of this type of tunable filter. It appears that filters can be constructed at frequencies lower and higher than X-band frequencies, and employing strip-transmission-line and coaxial-line coupling structures. Filters operating at lower frequencies (below S-band) will require a narrow-line-width magnetic material with a lower saturation magnetization than the yttrium-iron-garnet employed here. One material which appears to satisfy these requirements is gallium-substituted yttrium-iron-garnet.[34]

It appears possible to extend the tuning range of these filters beyond that reported here. This can presumably be accomplished by the use of ridged-waveguide, strip-transmission-line, and other inherently-wide-band coupling structures.

[34] E. G. Spencer and R. C. LeGraw, "Line width narrowing in gallium substituted yttrium iron garnet," *Bull. Am. Phys. Soc.*, Ser. 2, vol. 5, pt. 1, p. 58; January 27, 1960.

VII. Acknowledgment

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